Stat 217, Spring 2023 *Jiaze Qiu* (jiazeqiu@g.harvard.edu)

Section 5: Cavity Method for Spiked Matries

- Sections: Wed, 7:30-8:30pm (SC 705); OHs: Wed 8:30-9:30pm (SC 316.07).
- All the section materials (handouts & solutions) can be found either on Canvas or here.

1 Cavity method and message passing

- Notations and parametrizations: μ_i, $\mu_{i\to j}$, h_i, $h_{i\to j}$.
- Assumptions of the Cavity method
  1. Approximate independence among coordinates;
  2. $b_i := \frac{1}{n}\sum_{l\neq i} v_{0,l}\tanh(\beta h_{l\to i}) \approx b_\star$ and $q_i := \frac{1}{n}\sum_{l\neq i}\tanh^2(\beta h_{l\to i}) \approx q_\star$;
  3. If n is large enough, removing one (or any finite number of) coordinates should not change the system too mcuh.

1. Recall from lectures that, in order to carry out computation of the limiting free energy, we need to compute
$$\mathbb{E}\log\frac{Z_n(\beta_n,\lambda_n)}{Z_n(\beta,\lambda)} = T_1 + T_2 + o_p(1),$$
for which we have showed in lectures that as $n\to\infty$,
$$T_1 := \frac{\beta\lambda}{n^2}\sum_{i<j}\mathbb{E}\left[\sum_\sigma \mu(\sigma)\sigma_i\sigma_j\right] = \frac{\beta\lambda}{2}b_\star^2 + o_p(1),$$
and now we will try to "prove"
$$T_2 := \frac{\beta}{n^{3/2}}\sum_{i<j}\mathbb{E}\left[\tilde{W}_{ij}\sum_\sigma \mu(\sigma)\sigma_i\sigma_j\right] = \frac{\beta^2}{4}(1-q_\star^2) + o(1),$$
where $\tilde{W}_{ij} \overset{iid}{\sim} \mathcal{N}(0,1)$.

2. Recall that we also need to show $\frac{1}{n^2}\sum_{i<j}\sigma_i\sigma_j$ under the Gibbs measure μ. Note that since μ is a random measure, technically speaking, we need to be more careful but this is only a reminder and we won't worry it for now. Please try to justify it as best as you can.

3. Can you intuitively convince yourself why message passing is gauranteed to work on trees?

References

[1] Oliver Y Feng **andothers**. "A unifying tutorial on approximate message passing". **in***Foundations and Trends® in Machine Learning*: 15.4 (2022), **pages** 335–536.

[2] Charles W Fox **and** Stephen J Roberts. "A tutorial on variational Bayesian inference". **in***Artificial intelligence review*: 38 (2012), **pages** 85–95.

[3] Andrea Montanari **and** Subhabrata Sen. "A short tutorial on mean-field spin glass techniques for non-physicists". **in***arXiv preprint arXiv:2204.02909*: (2022).

[4] Dmitry Panchenko. *The sherrington-kirkpatrick model*. Springer Science & Business Media, 2013.